How to Efficiently Reach Largest Capacity with Fewest Relays?

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Introduction

Scenario

- Wireless communication
- **More relays → higher capacity**
- **Fewer relays → lower costs**
- Balance between capacity and cost

Problem Formulation

- When all relays work, the capacity reaches maximum (*global maximum capacity*). However, we **don't have to apply all relays** to reach *global maximum capacity*
- *Global network*: the subnetworks that reach *global maximum capacity*
- *Global optimal network*: the *global network* with fewest relays
- **Problem: how to find global optimal network efficiently?**

Existing Studies

- No directly related work found
- Calculation of capacity: [1] utilizes the submodularity property of the cut function and [2] proposes a method using dynamic programming approach
- Selection of relays: [2] applied simulated annealing approach, while [3] approximates the problem as a non-linear optimization problem
- Methods of existing studies are **time-consuming** for this problem

Objectives

Theory

- Explore structural characteristics of *global network*
- Design an explicit criterion to decide the *global optimal network*

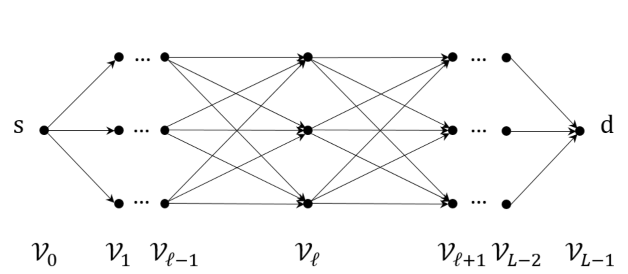
Practice

- Develop efficient algorithms to obtain the *global optimal network*
- Efficiently obtain the new result when parameters change

Model

Structure

- **Layered Gaussian relay network**
- Start from source s to destination d
- L layers, with K_ℓ nodes in the ℓth layer
- Every edge has a channel gain $H_{i,j}^{\ell}$
- $Y_j^{\ell+1} = \sum_{i=1}^{K_\ell} H_{i,j}^{l} X_i^{\ell} + Z_j^{\ell+1}$, $Z_j^{\ell+1}$ is i.i.d. $\mathcal{CN}(0,1)$



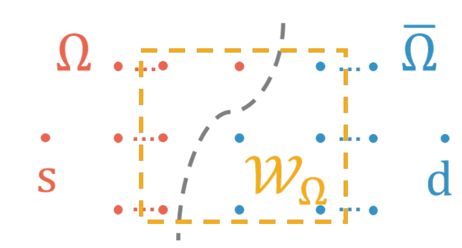
Capacity

- *Cut* Ω: a subset of the network that satisfies $s \in \Omega, d \in \bar{\Omega}$
- Approximate the cutset upper bound as the capacity
- For a given network $\mathcal{M}$, its capacity can be calculated as
$$C_{\mathcal{M}} = \min_{\{\Omega\}} \log\det\left(\mathbf{I} + \mathbf{H}_\Omega \mathbf{H}_\Omega^{\dagger}\right)$$
where $\mathbf{H}_\Omega$ denotes the MIMO channel matrix from nodes in Ω to nodes in $\bar{\Omega}$
- Use the characteristic of hierarchy, the equation can be reformulated as
$$C_{\mathcal{M}} = \min_{\{\Omega\}} \sum_{\ell=1}^{L-1} \log\det\left(\mathbf{I} + \mathbf{H}_\Omega^{\ell} \mathbf{H}_\Omega^{\ell\dagger}\right)$$
where $\mathbf{H}_\Omega^{\ell}$ denotes the MIMO channel matrix from nodes $\Omega \cap \mathcal{V}_{\ell-1}$ to nodes in $\bar{\Omega} \cap \mathcal{V}_\ell$

Theoretical Study

Definitions

Definition 1 Given a cut Ω and two nodes P,Q, if $P \in \Omega, Q \in \bar{\Omega}$ and Q is in the next layer of P, we define both P and Q as *working node.* Denote the set of working nodes as $\mathcal{W}_\Omega$



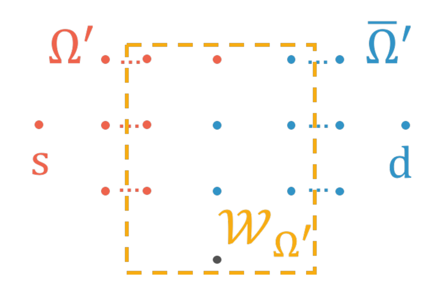
Definition 2 Define the cutset of a given cut Ω as
$$C(\Omega) = \log\det\left(\mathbf{I} + \mathbf{H}_\Omega \mathbf{H}_\Omega^{\dagger}\right)$$

Definition 3 Denote the **bottleneck cut** as $\Omega^* = \arg\min C(\Omega)$

Properties

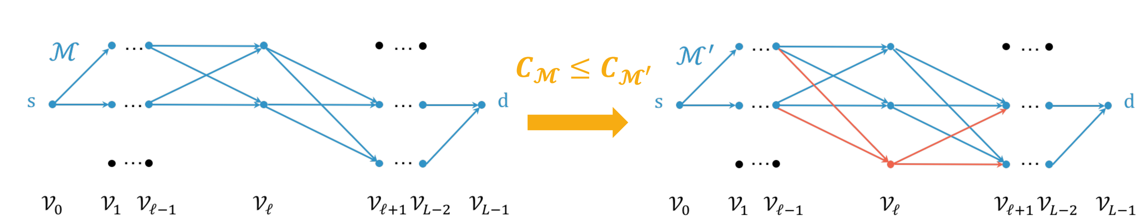
New nodes in $\mathcal{W}$

- Adding a new node into a $\mathcal{W}$ will strictly **increase** its cutset
- $C(\Omega') > C(\Omega)$



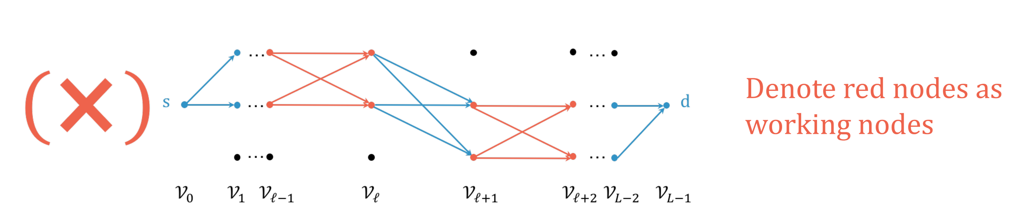
New nodes in $\mathcal{M}$

- When the node number of subnetwork increases, the capacity of the optimal subnetwork $\mathcal{M}^*$ will **not decrease**
- Adding new nodes into a subnetwork $\mathcal{M}$ will **not decrease** its capacity

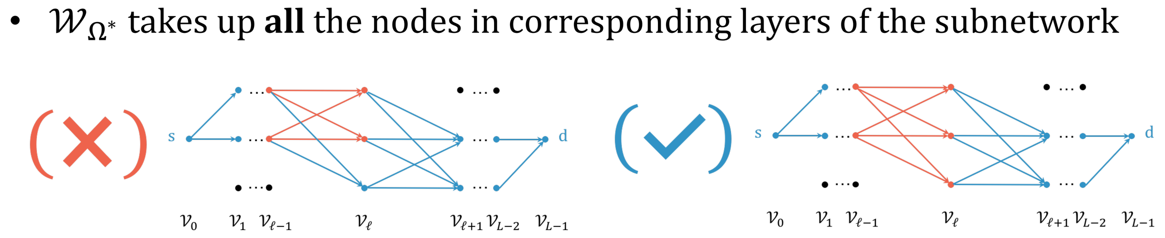


Bottleneck

- Layers of $\mathcal{W}_{\Omega^*}$ are **successive**

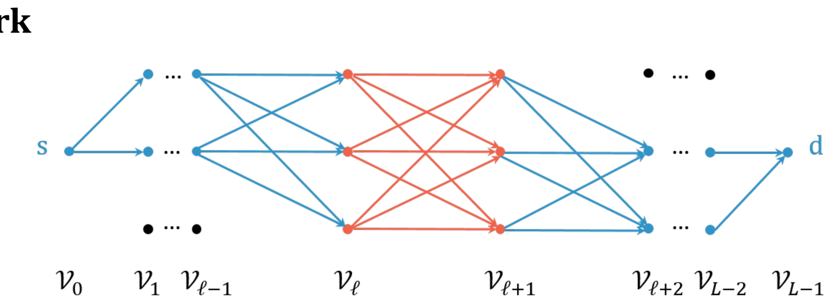


- $\mathcal{W}_{\Omega^*}$ takes up **all** the nodes in corresponding layers of the subnetwork



Criterion

A subnetwork is *global network* ⇔ $\mathcal{W}_{\Omega^*}$ takes up all the nodes in corresponding layers of the whole network



The subnetwork is *global network*

Remark

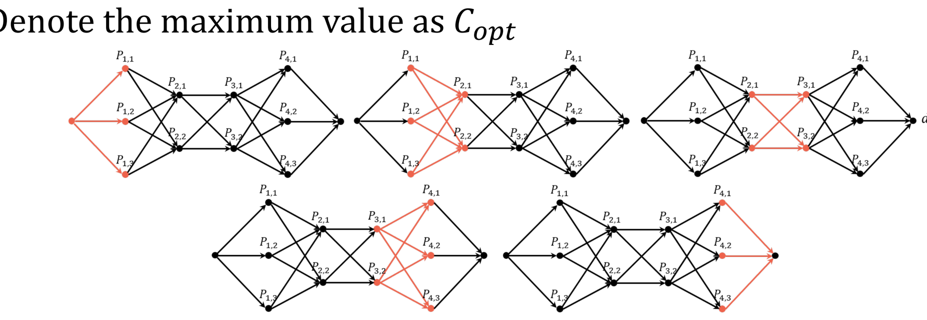
- Not necessary to calculate the whole network
- Reduce the time complexity of judgement by at least a half

Algorithms

Find global optimal network

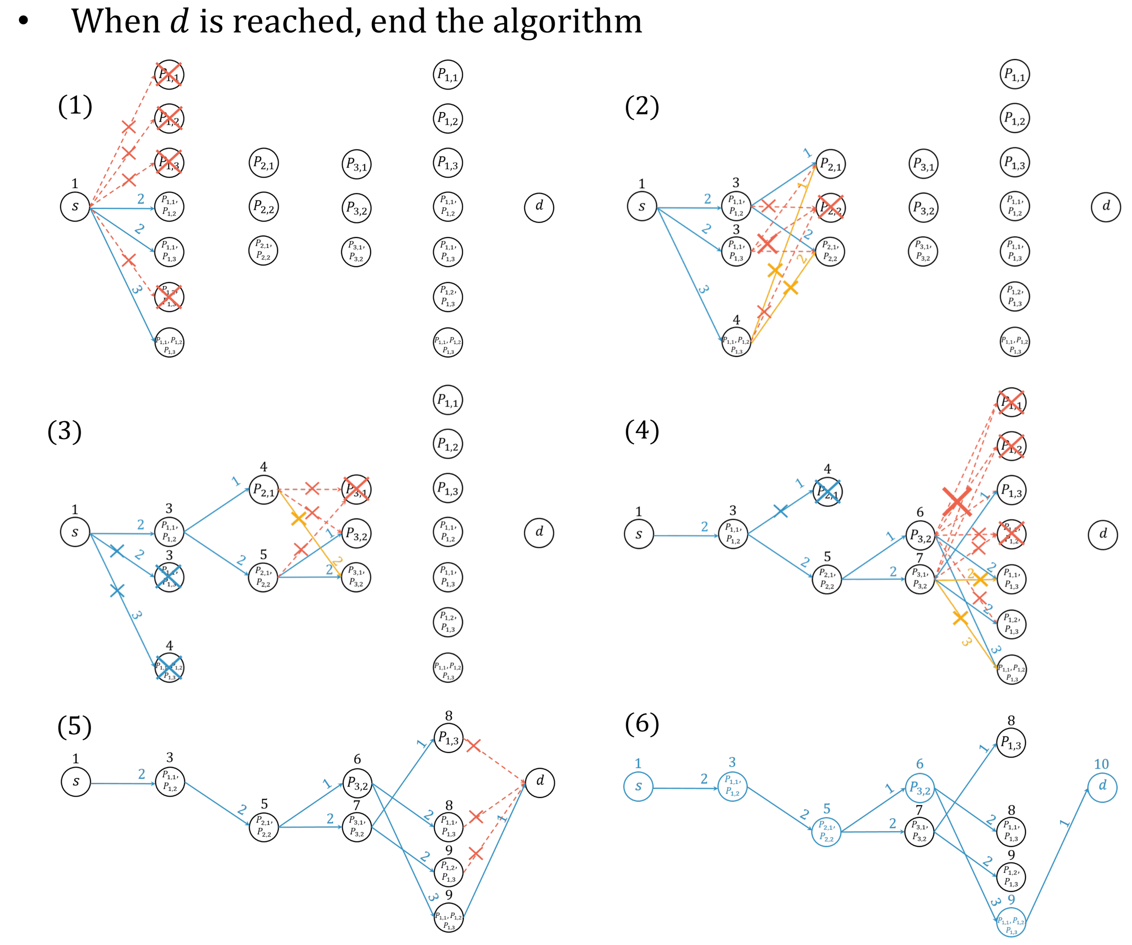
Step 1: Calculate global maximum capacity

- For every ℓ, calculate the **maximum cutset** between layer ℓ and layer ℓ+1
- Denote the maximum value as C_{opt}



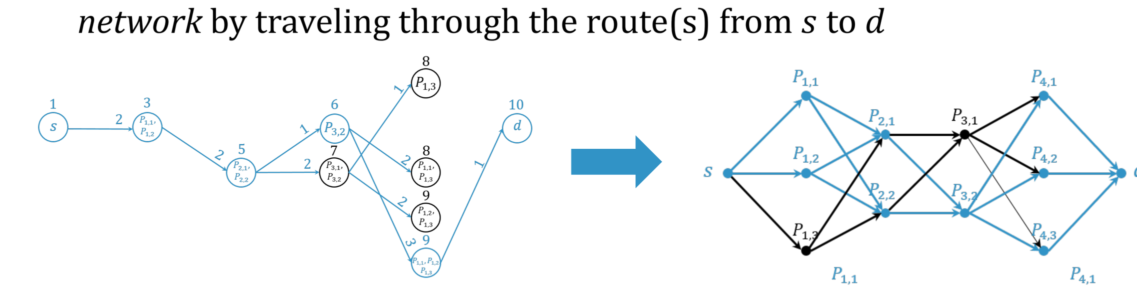
Step 2: Dynamic programming

- Draw the figure below, in which every circle represents a selection of nodes
- Calculate the cutset between possible pairs of circles respectively from **current** layer and **previous** layer
- Connect the circles whose cutset is **not larger than** C_{opt} (solid line)
- Delete nodes which has no connection with previous layer (marked with ×) or has nowhere to go (marked with ×)
- From s to the chosen circle in current layer, delete the paths that are not the **shortest** (marked with ×)
- Move to next layer
- When d is reached, end the algorithm



Step 3: Construct the global optimal network

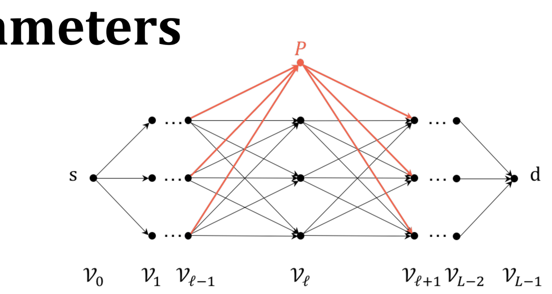
- From the result in Step 2, we can easily get the *global optimal network* by traveling through the route(s) from s to d



Deal with dynamic parameters

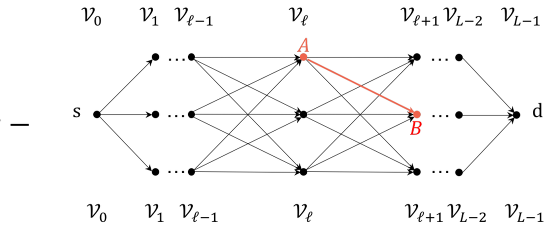
Adding new node P

- With the property of dynamic programming, only layer $\ell - 1, \ell$ and $\ell + 1$ need to be searched



Changing channel gain H_{AB}

- If H_{AB} satisfies
$$a|H_{AB}|^2 - 2|b||H_{AB}|\cos\left(\theta_{H_{AB}} - \theta_b\right) + c - \min\{d_1, d_2\} \geq 0^*$$
the result will **not change**



* Expressions of a, b, c, d_1, d_2 are omitted due to space limit

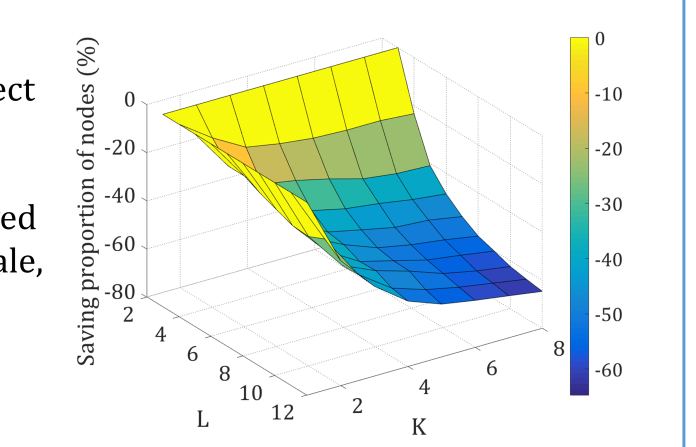
Results

Environment

- Individual channel gains were chosen to be i.i.d. according to $10\log_{10}\left(\left|H_{i,j}^{\ell}\right|^2\right) \sim U[0,35]$, $\angle H_{i,j}^{\ell} \sim U[0, 2\pi]$
- Test under MATLAB for Mac R2015b

Necessity

- When K and L grows, the effect becomes more significant
- When K = 8 and L = 12, near **70%** of the nodes can be saved
- In the network with large scale, only a fraction of nodes is needed to achieve *global maximum capacity*



Time complexity

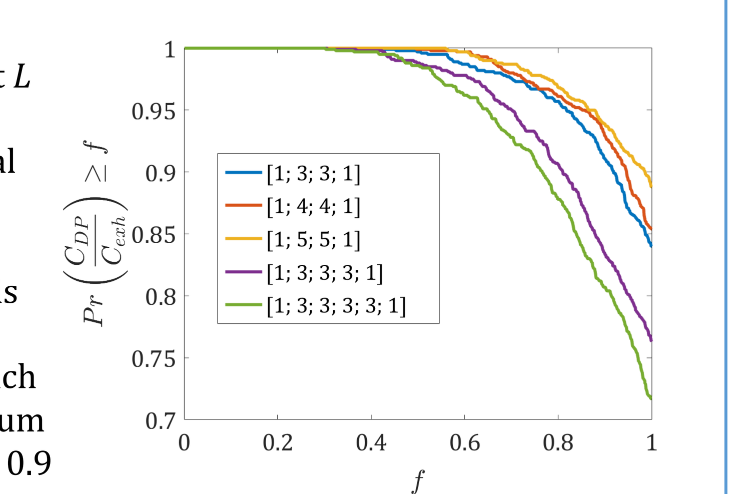
- Compare the time complexity between dynamic programming approach and exhaustive search approach
- With L (number of layers) and K (number of nodes in each middle layer) growing, the **acceleration multiple** grows **exponentially**

Table: Acceleration multiple of dynamic programming approach

	$L = 3$	$L = 4$	$L = 5$	$L = 6$
$K = 1$	0.96	1.01	1.02	0.97
$K = 2$	2.25	4.82	8.66	23.99
$K = 3$	4.32	9.12	53.12	363.09
$K = 4$	4.22	25.07	273.83	4910.98

Accuracy

- Test the cases of different L and K
- Generally can reach global maximum capacity with probability **0.8**
- Performs better when K is large
- In the worst case, can reach 80% of the global maximum capacity with probability 0.9



Conclusions

Network Properties

- We proposed and proved several properties of layered Gaussian relay network
- We strictly proved an theorem, which provides an **explicit criterion** to decide the *global optimal network*, not necessary to calculate through the whole network

Implementation

- We designed an efficient algorithm that utilizes dynamic programming approach and network properties which can **speed up** the solution **exponentially**
- We took advantage of the layered characteristic of the algorithm, and proposed methods that can calculate the new result efficiently (**warm start**) when parameters change

References

[1] F. Parvaresh and R. H. Etkin, "Efficient capacity computation and power optimization for relay networks," CoRR, vol. abs/1111.4244, 2011.

[2] Ritesh Kolte and Ayfer Özgür. Fast near-optimal subnetwork selection in layered relay networks. In Communication, Control, and Computing (Allerton), 2014 52nd Annual Allerton Conference on, pages 1238–1245. IEEE, 2014.

[3] Siddhartha Brahma, Ayan Sengupta, and Christina Fragouli. Efficient subnetwork selection in relay networks. In 2014 IEEE International Symposium on Information Theory, pages 1927–1931. Ieee, 2014.